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# COMPARABLE AND COMPLETE LATTICE STRUCTURE OF N-PICTURE FUZZY SOFT ENVIRONMENT

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***ABSTRACT:****In this study using the N-Picture fuzzy soft definitions, we define some concepts such as the N-Picture fuzzy soft lattice, N-Picture fuzzy soft sub lattice, complete N-Picture fuzzy soft lattice, modular N-Picture fuzzy soft lattice, distributive N-Picture fuzzy soft lattice, N-Picture fuzzy soft chain then we study the relationship and observe some common properties.*

***KEY WORDS:*** *Soft set, Fuzzy soft set, Picture fuzzy set, N-Picture fuzzy soft set, distributive lattice, modular lattice, chain.*

## 1. INTRODUCTION

Soft set theory [31] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertainty. The operations of soft sets are defined by Maji et al. [30] and redefined by Cagman and Enginoglu [6]. Recently, the properties and applications on the soft set theory have been studied increasingly [2, 9, 17, 36, 40]. The algebraic structure of soft set theory has also been studied in more detail [1, 4, 11, 18, 19, 21, 22, 23, 24, 25], and many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [4, 8, 12, 28, 37, 39]. The soft lattice structures are constructed by Nagarajan and Meenambigai [32] and Li [27] over a soft set. Manemaran and Nagarajan [[33], [34]] introduced N-Picture fuzzy soft (1,2)-ideal structures and temporal generated N-Picture fuzzy soft substructures. In this study using the N-Picture fuzzy soft definitions, we define some concepts such as the N-Picture fuzzy soft lattice, N-Picture fuzzy soft sub lattice, complete N-Picture fuzzy soft lattice, modular N-Picture fuzzy soft lattice, distributive N-Picture fuzzy soft lattice, N-Picture fuzzy soft chain then we study the relationship and observe some common properties.

## 2. BASIC CONCEPTS AND PRELIMINARIES

**Definition 2.1**: Let U be a non-empty set. Then by a fuzzy set on U is meant a functionA : U → [0, 1]. A is called the membership function, A(x) is called the membership grade of x in A. We also write A = {(x, A(x)) : x ∈ U}.

**Example 2.2:**Consider U = {x, y, z, p} and A : U → [0, 1] defined by A(x) = 0, A(y) = 0.1, A(z) = 0.9, A(p) = 1.

**Definition 2.3: [Naim cagman]** Let U be an initial universe, E be the set of all parameters and A⊆ E. A pair (F, A) is called a fuzzy soft set over U where F: A → $\tilde{P}$(U) is a mapping from A into $\tilde{P}$(U), where $\tilde{P}$(U) denotes the collection of all subsets of U.

**Example 2.4:**Consider the above example, here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1].Then

($f_A$, E) = {$f_A(e_1)$ = {($u_1$, 0.7), ($u_2$, 0.5), ($u_3$, 0.4), ($u_4$, 0.2)},

$f_A$ ($e_2$) = {($u_1$, 0.5), ($u_2$, 0.1), ($u_3$, 0.5)} is the fuzzy soft set representing the "colour of the shirts" which Mr. X is going to buy.

**Definition 2.5:**A Picture Fuzzy Set (PFS) A on a universe X is an object of the form A={(x,$\mu_A$(x),$\eta_A$(x),$\nu_A$(x))/x∈X}where $\mu_A$(x)∈[0,1] iscalled thedegree ofpositive membership(PM) of x in A, $\eta_A$(x)∈[0,1] is called the degree of neutral membership (NeuM) of x in A, $\nu_A$(x) ∈ [0,1] is called the degree of negative membership (NM) of x in A. $\mu_A$(x),$\eta_A$(x),$\nu_A$ (x) must satisfy the condition $\mu_A$(x)+$\eta_A$(x)+$\nu_A$(x) ≤1∀x ∈ X. Then∀x ∈ X, 1−($\mu_A$(x)+$\eta_A$(x)+$\nu_A$(x)) is called the degree of refusal membership of "x" in A.

**Definition 2.6:** A negative picture fuzzy (N-Picture fuzzy) soft set $\boldsymbol{\mathcal{A}}$ on the universe ofdiscourse$X$ is defined as $\boldsymbol{\mathcal{A}}$=$\langle x, \bar{\delta}_{\mathcal{A}}(x), \bar{I}_{\mathcal{A}}(x), \Delta_{\mathcal{A}}(x)\rangle$, $x \in \mathcal{X}$,where $\delta$, $I$, $\Delta$:$\boldsymbol{\mathcal{X}}$→[−1,0] and $-1 \leq \bar{\delta}_{\mathcal{A}}(x), \bar{I}_{\mathcal{A}}(x), \Delta_{\mathcal{A}}(x) \leq 0$.

**Definition 2.7:** A N-Picture fuzzy soft set$\boldsymbol{\mathcal{A}}$overtheuniverse $\boldsymbol{\mathcal{X}}$ is said to be null or empty N-Picture fuzzy soft setif $\delta_{\mathcal{A}}(x) = -1, I_{\mathcal{A}}(x) = -1, \Delta_{\mathcal{A}}(x) = 0$ for all $x \in \mathcal{X}$. It is denoted by $-1_N$.

**Definition 2.8:** A N-Picture fuzzy soft set$\mathcal{A}$overtheuniverse $\mathcal{X}$ is said to be absolute (universe) N-Picture fuzzy soft setif

$\delta_{\mathcal{A}}(x) = 0, I_{\mathcal{A}}(x) = 0, \Delta_{\mathcal{A}}(x) = -1$, for all $x \in \mathcal{X}$. It is denoted by $0_N$.

**Definition 2.9:** Let "M" be a non-empty ordered set.

(i) If $x \vee y$ and $x \wedge y$ exist for all $x,y \in M$, then M is called a Lattice.

(ii) If $\vee N$ and $\wedge N$ exists for all $N \subseteq M$, then M is called a Complete Lattice.

**Definition 2.10:** An algebra $(L, \vee, \wedge)$ is called a Lattice if L is a non-empty set, $\vee$ and $\wedge$ are binary operations on L, both $\vee$ and $\wedge$ are idempotent, commutative and associative and they satisfy the two absorption identities. That is, for all $a,b,c \in L$.

1. $a \wedge a = a$ ; $a \vee a = a$
2. $a \wedge b = b \wedge a$ ; $a \vee b = b \vee a$
3. $(a \wedge b) \wedge c = a \wedge (b \wedge c)$ ;$(a \vee b) \vee c = a \vee (b \vee c)$
4. $a \wedge (a \vee b) = a$ ; $a \vee (a \wedge b) = a$

**Definition 2.11:**Let $F_A$ and $G_B$ be two N-Picture fuzzy soft sets over the common universe U. $F_A$ is said to be a N-Picture fuzzy soft subset of $G_B$ if $A \subset B$ and $T_{F(c)}(x) \leq T_{G(c)}(x)$, $I_{F(c)}(x) \leq I_{G(c)}(x)$, $F_{F(c)}(x) \geq F_{G(c)}(x)$, for all $c \in A$, $x \in U$. We denote it by $F_A \subseteq G_B$.

**Definition 2.12:**The complement of a N-Picture fuzzy soft set $F_A$ denoted by $F_A^c$, when $F_A^c$: $\overline{A} \to P(U)$ is a mapping given by $F^c(\alpha)$ = N-Picture fuzzy soft set complement with $T_F^c(x) = F_F(x)$, $I_F^c(x) = I_F(x)$ and $F_F^c(x) = T_F(x)$.

**Definition 2.13:**Let $H_A$ and $G_B$ be two N-Picture fuzzy soft sets over the common universe U, then the intersection of $H_A$ and $G_B$ is denoted by $H_A \cap G_B$ and is defined by $H_A \cap G_B = K_c$, when $C = A \cap B$ and the truth-membership, indeterminacy-membership and falsify-membership of $K_c$ are as follows:

$$T_{K(c)}(m) = \min \{ T_{H(c)}(m), T_{G(c)}(m) \}$$

$$I_{K(c)}(m) = \frac{I_{H(c)}(m) + I_{G(c)}(m)}{2}$$

$$F_{K(c)}(m) = \max \{ F_{H(c)}(m), F_{G(c)}(m) \}, \text{ if } c \in A \cap B.$$

Also, $H_A \cup G_B = K_c$ where $c = A \cup B$ and the truth-membership, indeterminacy-membership and falsify-membership of $K_c$ are as follows:

$T_{K(c)}(m) = \max \{ T_{H(c)}(m), T_{G(c)}(m) \}$

$$I_{K(c)}(m) = \frac{I_{H(c)}(m) + I_{G(c)}(m)}{2}$$

$F_{K(c)}(m) = \min \{ F_{H(c)}(m), F_{G(c)}(m) \}$, if $c \in A \cap B$.

## 3. LATTICE STRUCTURE OF N-PICTURE FUZZY SOFT SETS

In this section, the notion of N-Picture fuzzy soft lattice is defined and several related properties are investigated.

**Definition 3.1:**Let $P^L$ be the N-Picture fuzzy soft set over U. $\vee$ and $\wedge$ be two binary operations on $P^L$. If elements of $P^L$ are equipped with two commutative and associative binary operations $\vee$ and $\wedge$ which are connected by the absorption law, then algebraic structure($P^L$, $\vee$ , $\wedge$ ) is called a N-Picture fuzzy soft set.

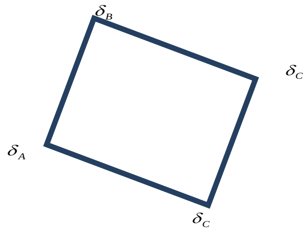


**Fig -1.Hasse diagram for N-Picture fuzzy soft latticestructure.**

**Example 3.2:** Let $U = \{u_1, u_2, u_3, u_4\}$ be a universe set and $P^L = \{\delta_A, \delta_B, \delta_C, \delta_D\} \subseteq$ PS(U).

Suppose that,

$$\delta_A = \left\{ \left( e_1, \frac{u_1}{-0.3,-0.2,-0.7} \right), \left( e_2, \frac{u_2}{-0.3,-0.6,-0.3}, \frac{u_3}{-0.3,-0.7-0.8} \right), \left( e_3, \frac{u_4}{-0.2,-0.6,-0.7} \right) \right\}$$

$$\delta_B = \left\{ \left( e_1, \frac{u_1}{-0.1,-0.4,-0.6}, \frac{u_2}{-0.2,-0.6,-0.8} \right), \left( e_2, \frac{u_3}{-0.4,-0.6-0.7} \right) \right\}$$

$$\delta_C = \left\{ \left( e_1, \frac{u_1}{-0.4,-0.2,-0.5}, \frac{u_2}{-0.1,-0.5,-0.7} \right), \left( e_2, \frac{u_2}{-0.4,-0.6,-0.2}, \frac{u_3}{-0.4,-0.7,-0.8} \right) \right\}$$

$$\delta_D = \left\{\left(e_1, \frac{u_1}{-0.7,-0.6,-0.4}, \frac{u_2}{-0.6,-0.2,-0.3}\right)\right\}$$

Then $(P^L,\cup,\cap)$ is a N-Picture fuzzy soft lattices.

**Proposition 3.3:** Let $(P^L,\vee,\wedge)$ be a N-Picture fuzzy soft lattice and $\delta_A,\delta_B \in$ PS(U), then $\delta_A \wedge \delta_B = \delta_A$ if and only if $\delta_A \vee \delta_B = \delta_B$.

**Proof:** Here $(P^L,\vee,\wedge)$ be a N-Picture fuzzy soft lattice and $\delta_A$, $\delta_B \in$ PS(U).

Now, $\delta_A \wedge \delta_B = \delta_A \wedge (\delta_A \vee \delta_B)$

$= (\delta_A \wedge \delta_A) \wedge (\delta_A \wedge \delta_B)$ by distribution lattice

$= \delta_A \wedge \delta_A$

$= \delta_A$

Conversely, $\delta_A \vee \delta_B = (\delta_A \wedge \delta_B) \vee \delta_B$

$= (\delta_A \vee \delta_B) \wedge (\delta_B \vee \delta_B)$ by distribution lattice

$= \delta_B \wedge \delta_B$

$= \delta_B$. Hence, the proof.

**Proposition 3.4:** Let $(P^L,\vee,\wedge)$ be a N-Picture fuzzy soft lattice and $\delta_A,\delta_B \in$ PS(U), then the relation $\leq$ which is defined by $\delta_A \leq \delta_B$ if and only if $\delta_A \wedge \delta_B = \delta_A$ or $\delta_A \vee \delta_B = \delta_B$.

**Proof:** (i) the order relation is reflexive, for all $\delta_A \in P^L$,

Let $\delta_A \leq \delta_B$ if and only if $\delta_A \wedge \delta_B = \delta_A$.

(ii) the order relation is anti-symmetric, for all $\delta_A$, $\delta_B \in P^L$,

Let $\delta_A \leq \delta_B$ and $\delta_B \leq \delta_A$.

Then, $\delta_A = \delta_A \wedge \delta_B$

$= \delta_B \wedge \delta_A$

$= \delta_B$

(iii) the order relation is transitive, for all $\delta_A$, $\delta_B$ and $\delta_C \in P^L$,

If $\delta_A \leq \delta_B$ and $\delta_B \leq \delta_C$ implies $\delta_A \leq \delta_C$

Indeed $\delta_A \wedge \delta_B = (\delta_A \wedge \delta_B) \wedge \delta_C$

$= (\delta_A) \wedge (\delta_B \wedge \delta_C)$

$= \delta_A \wedge \delta_B$

$= \delta_A$. The theorem is proved.

**Proposition 3.5:** Let ($P^L$, $\vee$, $\wedge$) be a N-Picture fuzzy soft lattice and $\delta_A, \delta_B \in$ PS(U), then $\delta_A \vee \delta_B$ and $\delta_A \wedge \delta_B$ on the least upper and greatest lower bound of $\delta_A$ and $\delta_B$ respectively.

**Proof:** Suppose that $\delta_A \wedge \delta_B$ is not the greatest lower bound of $\delta_A$ and $\delta_B$, then there exists $\delta_C \in$ PS(U), such that

$\delta_A \wedge \delta_B \leq \delta_C \leq \delta_A$ and

$\delta_A \wedge \delta_B \leq \delta_C \leq \delta_B$.

Hence, $\delta_C \wedge \delta_C \leq \delta_A \wedge \delta_B$.

Thus, $\delta_C \leq \delta_A \wedge \delta_B$.

Therefore, $\delta_C = \delta_A \wedge \delta_B$.But this is contradiction.

$\delta_A \vee \delta_B$ being the least upper bound of $\delta_A$ and $\delta_B$ can be shown in a similar way.

**Lemma 3.6:** Let $P^L \in$ PS(U). Then N-Picture fuzzy soft lattice inclusion relation '$\subseteq$' that is defined by $\delta_A \subseteq \delta_B$ if and only if $\delta_A \cup \delta_B = \delta_B$ or $\delta_A \cap \delta_B = \delta_A$ is an ordering relation on $P^L$.

**Proof:**For all $\delta_A$, $\delta_B$ and $\delta_C \in P^L$,

(i) $\delta_A \in P^L$, $\subseteq$ is reflexive, $\delta_A \subseteq \delta_A \Leftrightarrow \delta_A \cap \delta_A = \delta_A$

(ii) $\delta_A$, $\delta_B \in P^L$, $\subseteq$ is anti-symmetric, let $\delta_A \subseteq \delta_B$ and $\delta_B \subseteq \delta_A \Leftrightarrow \delta_A = \delta_B$

(iii) $\delta_A$, $\delta_B$ and $\delta_C \in P^L$, $\subseteq$ is transitive, If $\delta_A \subseteq \delta_B$ and $\delta_B \subseteq \delta_C \Leftrightarrow \delta_A \subseteq \delta_C$

**Corollary 3.7:** ($P^L$, $\cup$, $\cap$, $\subseteq$) is a N-Picture fuzzy soft lattice.

**Definition 3.8:** Let ($P^L$, $\vee$, $\wedge$, $\leq$) be a N-Picture fuzzy soft lattice and let $\delta_A \in P^L$. If $\delta_A \leq \delta_B$, for all $\delta_B \in P^L$, then "$\delta_A$" is called the minimal element of $P^L$. If $\delta_B \leq \delta_A$, for all $\delta_B \in P^L$, then "$\delta_A$" is called the maximal element of $P^L$.

**Definition 3.9:** Let ($P^L$, $\vee$, $\wedge$, $\leq$) be a N-Picture fuzzy soft lattice and let $\delta_A \in P^L$. If $\delta_B \leq \delta_A$ or $\delta_A \leq \delta_B$, for all $\delta_A$, $\delta_B \in P^L$, then "$P^L$" is called a N-Picture fuzzy soft chain.

**Example 3.10:** Consider the N-Picture fuzzy soft lattice in Example-1. A N-Picture fuzzy soft subset $P^S = \{\delta_A, \delta_B, \delta_D\} \subseteq$ PS(U) of $P^L$ is a N-Picture fuzzy soft chain. But ($P^L$, $\cup$, $\cap$, $\subseteq$) is not a N-Picture fuzzy soft chain. Since, $\delta_B$ and $\delta_C$ cannot be comparable.

**Definition 3.11:** Let ($P^L$, $\vee$, $\wedge$, $\leq$) be a N-Picture fuzzy soft lattice. If every subset of $P^L$ have both a greatest lower bound and a least upper bound, the $P^L$ is called a complete N-Picture fuzzy soft lattice.

**Example 3.12:** Let $U=\{u_1,u_2,u_3,u_4\}$ be a universe set and $P^L = \{\delta_A,\delta_B,\delta_C, \delta_D\} \subseteq$ PS(U).

Suppose that,

$$\delta_A = \left\{\left(e_1, \frac{u_1}{-0.7,-0.5,-0.3}, \frac{u_4}{-0.2,-0.8,-0.3}\right)\right\}$$

$$\delta_B = \left\{\left(e_1, \frac{u_1}{-0.1,-0.3,-0.7}, \frac{u_3}{-0.3,-0.5,-0.7}\right), \left(e_2, \frac{u_2}{-0.4,-0.6,-0.7}, \frac{u_1}{-0.7,-0.8,-0.4}\right)\right\}$$

$$\delta_C = \left\{\left(e_1, \frac{u_1}{-0.4,-0.3,-0.6}, \frac{u_2}{-0.3,-0.6,-0.1}, \frac{u_3}{-0.4,-0.7,-0.8}, \frac{u_4}{-0.7,-0.5,-0.4}\right)\right\}$$

$$\delta_D = \phi$$

then ($P^L$, $\cup$, $\cap$, $\subseteq$) is a complete N-Picture fuzzy soft lattice.

**Definition 3.13:** Let ($P^L$, $\vee$, $\wedge$, $\leq$) be a N-Picture fuzzy soft lattice and $P^R \subseteq P^L$. If $P^R$ is N-Picture fuzzy soft lattice with the operation of $P^L$, then $P^R$ is a N-Picture fuzzy soft sub lattice of $P^L$.

**Proof:** It is obvious from Definition.

**Example 3.14:** Let $U=\{u_1,u_2,u_3,u_4\}$ be a Universe set and $P^L = \{\delta_A,\delta_B,\delta_C, \delta_D\} \subseteq$ PS(U).

Suppose that,

$$\delta_A = \left\{\left(e_1, \frac{u_1}{-0.7,-0.6,-0.3}, \frac{u_3}{-0.6,-0.4,-0.2}\right)\right\}$$

$$\delta_B = \left\{\left(e_1, \frac{u_1}{-0.2,-0.1,-0.3}, \frac{u_3}{-0.4,-0.1,-0.6}, \frac{u_5}{-0.1,-0.7,-0.6}\right), \left(e_2, \frac{u_2}{-0.4,-0.6,-0.3}, \frac{u_4}{-0.1,-0.7,-0.4}\right)\right\}$$

$$\delta_C = \left\{\left(e_1, \frac{u_1}{-0.4,-0.2,-0.5}, \frac{u_2}{-0.3,-0.4,-0.6}, \frac{u_5}{-0.6,-0.1,-0.7}\right), \left(e_2, \frac{u_1}{-0.7,-0.3,-0.4}, \frac{u_3}{-0.3,-0.6,-0.1}\right)\right\}$$

$\delta_D = \phi$

If $P^R = \{\delta_A, \delta_B, \delta_D\} \subseteq$ PS(U) then $P^R$ is N-Picture fuzzy soft sub lattice.

**Definition 3.15:** Let ($P^L, \vee, \wedge, \leq$) be a N-Picture fuzzy soft lattice and $\delta_A, \delta_B, \delta_C \in P^L$. If $(\delta_A \wedge \delta_B) \vee (\delta_A \wedge \delta_C) \leq \delta_A \wedge (\delta_B \vee \delta_C)$ or $\delta_A \wedge (\delta_B \vee \delta_C) \leq (\delta_A \wedge \delta_B) \vee (\delta_A \wedge \delta_C)$ then $P^L$ is called a one-sided distributive N-Picture fuzzy soft lattice.

**Theorem 3.16:**Every N-Picture fuzzy soft lattice is a one-sided distributed N-Picture fuzzy soft lattice.

**Proof:** Let $\delta_A, \delta_B$ and $\delta_C \in P^L$

Since, $\delta_A \wedge \delta_B \leq \delta_A$ and $\delta_A \wedge \delta_B \leq \delta_B \leq \delta_B \wedge \delta_C$,

$\delta_A \wedge \delta_B \leq \delta_A$ and $\delta_A \wedge \delta_B \leq \delta_B \wedge \delta_C$

Therefore,

$$\delta_A \wedge \delta_B = (\delta_A \wedge \delta_B) \wedge (\delta_A \wedge \delta_B) \leq \delta_A \wedge (\delta_B \vee \delta_C) \qquad \text{---------- (1)}$$

And also we have,

$\delta_A \wedge \delta_C \leq \delta_A$ and $\delta_A \wedge \delta_C \leq \delta_C \leq \delta_B \vee \delta_C$

Since, $\delta_A \wedge \delta_C \leq \delta_A$ and $\delta_A \wedge \delta_C \leq \delta_B \vee \delta_C$, then

$$\delta_A \wedge \delta_C = (\delta_A \wedge \delta_C) \wedge (\delta_A \wedge \delta_C) \leq \delta_A \wedge (\delta_B \vee \delta_C) \qquad \text{---------- (2)}$$

From (1) and (2), we have

$$(\delta_A \wedge \delta_B) \vee (\delta_A \wedge \delta_C) \leq \delta_A \wedge (\delta_B \vee \delta_C).$$

**Definition 3.17:** Let($P^L, \vee, \wedge, \leq$) be a N-Picture fuzzy soft lattice. If $P^L$ satisfies the following axioms, it is called a distributive N-Picture fuzzy soft lattice;

(i) $\delta_A \vee (\delta_B \wedge \delta_C) = (\delta_A \vee \delta_B) \wedge (\delta_A \vee \delta_C)$

(ii) $\delta_A \wedge (\delta_B \vee \delta_C) = (\delta_A \wedge \delta_B) \vee (\delta_A \wedge \delta_C)$, for all $\delta_A, \delta_B$ and $\delta_C \in P^L$.

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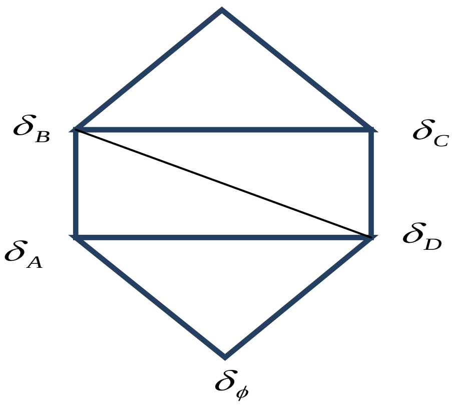


**Fig-2. Hasse diagram for distributive N-Picture fuzzy soft lattice structure**

**Example 3.18:** Let $U=\{u_1,u_2,u_3,u_4,u_5\}$ be a universe set and $P^L=\{\delta_\phi, \delta_A,\delta_B,\delta_C, \delta_D, \delta_E\}\subseteq$ PS(U). Then $P^L\subseteq$ PS(U) Is a N-Picture fuzzy soft lattice with operations $\cup$ and $\cap$.

Suppose that,

$$\delta_A=\left\{\left(e_1,\frac{u_1}{-0.7,-0.6,-0.3},\frac{u_3}{-0.6,-0.4,-0.2}\right)\right\}$$

$$\delta_B=\left\{\left(e_1,\frac{u_1}{-0.2,-0.1,-0.3},\frac{u_3}{-0.4,-0.1,-0.6},\frac{u_5}{-0.1,-0.7,-0.6}\right),\left(e_2,\frac{u_2}{-0.4,-0.6,-0.3},\frac{u_4}{-0.1,-0.7,-0.4}\right)\right\}$$

$$\delta_C=\left\{\left(e_1,\frac{u_1}{-0.4,-0.2,-0.5},\frac{u_2}{-0.3,-0.4,-0.6},\frac{u_5}{-0.6,-0.1,-0.7}\right),\left(e_2,\frac{u_1}{-0.7,-0.3,-0.4},\frac{u_3}{-0.3,-0.6,-0.1}\right)\right\}$$

$$\delta_D=\phi$$

Therefore, ($P^L$, $\cup$, $\cap$, $\subseteq$) is a distributive N-Picture fuzzy soft lattice.

**Definition 3.19:** Let($P^L$, $\vee$, $\wedge$, $\leq$) be a N-Picture fuzzy soft lattice, then $P^L$ is called a N-Picture fuzzy soft Module Lattice, if it is satisfies the following property;

$$\delta_C\leq\delta_A\Rightarrow\delta_A\wedge(\delta_B\vee\delta_C)=(\delta_A\wedge\delta_B)\vee\delta_C, \text{ for all } \delta_A,\delta_B \text{ and } \delta_C\in P^L.$$

**Example 3.20:**Let $U=\{u_1,u_2,u_3,u_4,u_5\}$ be a universe set and $P^L=\{\delta_\phi, \delta_A,\delta_B,\delta_C, \delta_D, \delta_E\}\subseteq$ PS(U). Then $P^L\subseteq$ PS(U) Is a N-Picture fuzzy soft lattice with operations $\cup$ and $\cap$.

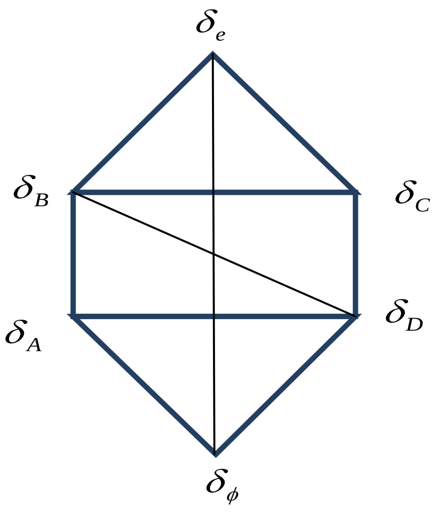


**Fig-3. Hasse diagram for modular N-Picture fuzzy soft latticestructure.**

Suppose that,

$$\delta_A = \left\{ \left( e_1, \frac{u_1}{-0.7,-0.6,-0.3}, \frac{u_3}{-0.6,-0.4,-0.2} \right) \right\}$$

$$\delta_B = \left\{ \left( e_1, \frac{u_1}{-0.2,-0.1,-0.3}, \frac{u_3}{-0.4,-0.1,-0.6}, \frac{u_5}{-0.1,-0.7,-0.6} \right), \left( e_2, \frac{u_2}{-0.4,-0.6,-0.3}, \frac{u_4}{-0.1,-0.7,-0.4} \right) \right\}$$

$$\delta_C = \left\{ \left( e_1, \frac{u_1}{-0.4,-0.2,-0.5}, \frac{u_2}{-0.3,-0.4,-0.6}, \frac{u_5}{-0.6,-0.1,-0.7} \right), \left( e_2, \frac{u_1}{-0.7,-0.3,-0.4}, \frac{u_3}{-0.3,-0.6,-0.1} \right) \right\}$$

$$\delta_D = \phi$$

Therefore, ($P^L$, $\cup$, $\cap$, $\subseteq$) is a modular N-Picture fuzzy soft lattice.

## 4. CONCLUSION

In this paper, we defined the concept of N-Picture fuzzy soft lattice ad an algebraic structure and showed that these definitions are equivalent. We then investigated some related properties and some important characterization theorems.

**Open problem:** A necessary and sufficient condition in order that the lattice of N-Picture fuzzy soft set is distributive (or modular).

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